

## Chapter 7

Quantum Theory and Atomic Structure

# Quantum Theory and Atomic Structure 

### 7.1 The Nature of Light

### 7.2 Atomic Spectra

7.3 The Wave-Particle Duality of Matter and Energy
7.4 The Quantum-Mechanical Model of the Atom

## The Wave Nature of Light

Visible light is a type of electromagnetic radiation.
The wave properties of electromagnetic radiation are described by three variables:

- frequency (v), cycles per second
- wavelength ( $\lambda$ ), the distance a wave travels in one cycle
- amplitude, the height of a wave crest or depth of a trough.

The speed of light is a constant:

$$
c=v \times \lambda
$$

$=3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ in a vacuum

Figure 7.1 The reciprocal relationship of frequency and wavelength.


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Figure 7.2 Differing amplitude (brightness, or intensity) of a wave.


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## Figure 7.3 Regions of the electromagnetic spectrum.



## Sample Problem 7.1 Interconverting Wavelength and Frequency

PROBLEM: A dental hygienist uses $x$-rays ( $\lambda=1.00 \AA$ ) to take a series of dental radiographs while the patient listens to a radio station ( $\lambda=325 \mathrm{~cm}$ ) and looks out the window at the blue sky ( $\lambda=$ 473 nm ). What is the frequency (in s${ }^{-1}$ ) of the electromagnetic radiation from each source? (Assume that the radiation travels at the speed of light, $3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$.)
PLAN: Use the equation $c=v \lambda$ to convert wavelength to frequency. Wavelengths need to be in meters because $c$ has units of $\mathrm{m} / \mathrm{s}$.

## wavelength in units given

use conversion factors
$1 \AA=10^{-10} \mathrm{~m}$
wavelength in m

$$
v=\frac{c}{\lambda}
$$

frequency ( $\mathrm{s}^{-1}$ or Hz )

## Sample Problem 7.1

## SOLUTION:

For the x-rays: $\quad \lambda=1.00 \AA \times \frac{10^{-10} \mathrm{~m}}{1 \AA}=1.00 \times 10^{-10} \mathrm{~m}$

$$
v=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1.00 \times 10^{-10} \mathrm{~m}}=3.00 \times 10^{18} \mathrm{~s}^{-1}
$$

For the radio signal: $\quad v=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{325 \mathrm{~cm} \times \frac{10^{-2} \mathrm{~m}}{1 \mathrm{~cm}}} \quad=9.23 \times 10^{7} \mathrm{~s}^{-1}$

For the blue sky: $\quad v=\frac{c}{\lambda}=\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{473 \mathrm{~nm} \times \frac{10^{-9} \mathrm{~m}}{1 \mathrm{~cm}}} \quad=6.34 \times 10^{14} \mathrm{~s}^{-1}$

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Figure 7.4 Different behaviors of waves and particles.


Figure 7.5 Formation of a diffraction pattern.


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## Energy and frequency

A solid object emits visible light when it is heated to about 1000 K . This is called blackbody radiation.

The color (and the intensity ) of the light changes as the temperature changes. Color is related to wavelength and frequency, while temperature is related to energy.

Energy is therefore related to frequency and wavelength:

$$
E=n h v
$$

$E=$ energy
$n$ is a positive integer $h$ is Planck's constant

## The Quantum Theory of Energy

## Any object (including atoms) can emit or absorb only certain quantities of energy.

Energy is quantized; it occurs in fixed quantities, rather than being continuous. Each fixed quantity of energy is called a quantum.

An atom changes its energy state by emitting or absorbing one or more quanta of energy.
$\Delta E=n h \nu$ where $n$ can only be a whole number.

Figure 7.6 The photoelectric effect.


## Sample Problem 7.2 Calculating the Energy of Radiation from Its Wavelength

PROBLEM: A cook uses a microwave oven to heat a meal. The wavelength of the radiation is 1.20 cm . What is the energy of one photon of this microwave radiation?

PLAN: We know $\lambda$ in cm , so we convert to $m$ and find the frequency using the speed of light. We then find the energy of one photon using $E=h v$.

## SOLUTION:

$E=h v=\frac{h c}{\lambda}=\frac{\left.\left(6.626 \times 10^{-34}\right) \mathrm{J} \cdot \mathrm{s}\right)\left(3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}{(1.20 \mathrm{~cm})\left(\frac{10^{-2} \mathrm{~m}}{1 \mathrm{~cm}}\right)}=1.66 \times 10^{-23} \mathrm{~J}$

Figure 7.7A The line spectrum of hydrogen.


Figure 7.7B $\quad$ The line spectra of Hg and Sr .


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Figure 7.8 Three series of spectral lines of atomic hydrogen.


Rydberg equation $\frac{1}{\lambda}=R\left(\frac{1}{n_{1}{ }^{2}}-\frac{1}{n_{2}{ }^{2}}\right)$
$R$ is the Rydberg constant $=1.096776 \times 10^{7} \mathrm{~m}^{-1}$
for the visible series, $n_{1}=2$ and $n_{2}=3,4,5, \ldots$

## The Bohr Model of the Hydrogen Atom

Bohr's atomic model postulated the following:

- The H atom has only certain energy levels, which Bohr called stationary states.
- Each state is associated with a fixed circular orbit of the electron around the nucleus.
- The higher the energy level, the farther the orbit is from the nucleus.
- When the H electron is in the first orbit, the atom is in its lowest energy state, called the ground state.

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- The atom does not radiate energy while in one of its stationary states.
- The atom changes to another stationary state only by absorbing or emitting a photon.
- The energy of the photon ( $h v$ ) equals the difference between the energies of the two energy states.
- When the E electron is in any orbit higher than $n=1$, the atom is in an excited state.

Figure 7.9 A quantum "staircase" as an analogy for atomic energy levels.


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Figure 7.10 The Bohr explanation of three series of spectral lines emitted by the H atom.


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A tabletop analogy for the H atom's energy.


$$
\Delta E=E_{\text {final }}-E_{\text {initial }}=-2.18 \times 10^{-18} \mathrm{~J}\left(\frac{1}{n_{\text {tinal }}^{2}}-\frac{1}{n_{\text {initial }}^{2}}\right)
$$

PROBLEM: A hydrogen atom absorbs a photon of UV light (see Figure 7.10) and its electron enters the $n=4$ energy level. Calculate (a) the change in energy of the atom and (b) the wavelength (in nm ) of the photon.

PLAN: (a) The H atom absorbs energy, so $E_{\text {final }}>E_{\text {initial }}$. We are given $n_{\text {final }}=4$, and Figure 7.10 shows that $n_{\text {initial }}=1$ because a UV photon is absorbed. We apply Equation 7.4 to find $\Delta E$.
(b) Once we know $\Delta E$, we find frequency and wavelength.

## Sample Problem 7.3

SOLUTION:
(a) $\quad \Delta E=-2.18 \times 10^{-18} \mathrm{~J}\left(\frac{1}{n_{\text {final }}^{2}}-\frac{1}{n_{\text {initial }}^{2}}\right)=-2.18 \times 10^{-18} \mathrm{~J}\left(\frac{1}{4^{2}}-\frac{1}{1^{2}}\right)$
$=-2.18 \times 10^{-18} \mathrm{~J}\left(\frac{1}{16}-\frac{1}{4}\right)=2.04 \times 10^{-18} \mathrm{~J}$
(b) $\lambda=\frac{h c}{\Delta E}=\frac{\left(6.626 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}\right)\left(3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)}{2.04 \times 10^{-18} \mathrm{~J}}=9.74 \times 10^{-8} \mathrm{~m}$

$$
9.74 \times 10^{-8} \mathrm{~m} \times \frac{1 \mathrm{~nm}}{10^{-9} \mathrm{~m}}=97.4 \mathrm{~nm}
$$

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Figure 7.11 Measuring chlorophyll a concentration in leaf extract.


## The Wave-Particle Duality of Matter and Energy

Matter and Energy are alternate forms of the same entity.

$$
E=m c^{2}
$$

All matter exhibits properties of both particles and waves. Electrons have wave-like motion and therefore have only certain allowable frequencies and energies.

Matter behaves as though it moves in a wave, and the de Broglie wavelength for any particle is given by:

$$
\lambda=\frac{h}{m u} \quad \begin{array}{ll}
m=\text { mass } \\
u=\text { speed in } \mathrm{m} / \mathrm{s}
\end{array}
$$

Figure $7.12 \quad$ Wave motion in restricted systems.


Table 7.1 The de Broglie Wavelengths of Several Objects

| Substance | Mass $(\mathbf{g})$ | Speed $(\mathrm{m} / \mathbf{s})$ | $\lambda(\mathbf{m})$ |
| :--- | :---: | :---: | :---: |
| slow electron | $9 \times 10^{-28}$ | 1.0 | $7 \times 10^{-4}$ |
| fast electron | $9 \times 10^{-28}$ | $5.9 \times 10^{6}$ | $1 \times 10^{-1}$ |
| alpha particle | $6.6 \times 10^{-24}$ | $1.5 \times 10^{7}$ | $7 \times 10^{-1}$ |
| one-gram mass | 1.0 | 0.01 | $7 \times 10^{-29}$ |
| baseball | 142 | 25.0 | $2 \times 10^{-34}$ |
| Earth | $6.0 \times 10^{27}$ | $3.0 \times 10^{4}$ | $4 \times 10^{-63}$ |

Sample Problem 7.4 Calculating the de Broglie Wavelength of an Electron

PROBLEM: Find the de Broglie wavelength of an electron with a speed of $1.00 \times 10^{6} \mathrm{~m} / \mathrm{s}$ (electron mass $=9.11 \times 10^{-31} \mathrm{~kg}$; $\left.h=6.626 \times 10^{-34} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}\right)$.

PLAN: We know the speed and mass of the electron, so we substitute these into Equation 7.5 to find $\lambda$.

SOLUTION: $\lambda=\frac{h}{m u}$

$$
\lambda=\frac{6.626 \times 10^{-34} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}}{9.11 \times 10^{-31} \mathrm{~kg} \times 1.00 \times 10^{6} \mathrm{~m} / \mathrm{s}} \quad=7.27 \times 10^{-10} \mathrm{~m}
$$

Figure 7.13 Diffraction patterns of aluminum with x-rays and electrons.

$x$-ray diffraction of aluminum foil

electron diffraction of aluminum foil

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Figure 7.14
Major observations and theories leading from classical theory to quantum theory

| Since matter is discontinuous and particulate, <br> perhaps energy is discontinuous and particulate. |  |
| :--- | :--- |
| Observation <br> Theory |  |
| Blackbody radiation $\longrightarrow$ Planck:Energy is quantized; only certain values <br> allowed |  |
| Atomic line spectra $\longrightarrow$ Bohr:Energy of atoms is quantized; photon <br> emitted when electron changes orbit. |  |

Figure 7.14 continued

|  | Since energy is wavelike, <br> perhaps matter is wavelike. |
| :--- | :--- |
| Observation <br> Davisson/Germer: <br> Electron beam is <br> diffracted by metal <br> crystal | Theory <br> deBroglie: All matter travels in waves; energy of <br> atom is quantized due to wave motion of <br> electrons |
|  | Since matter has mass, <br> perhaps energy has mass |
| Observation <br> Theory |  |
| Compton: Photon's <br> wavelength increases <br> (momentum decreases) <br> after colliding with <br> electron | Einstein/deBroglie: Mass and energy are <br> equivalent; particles have <br> wavelength and photons have <br> momentum. |
|  | QUANTUM THEORY <br> Energy and Matter <br> particulate, massive, wavelike |

## Heisenberg's Uncertainty Principle

Heisenberg's Uncertainty Principle states that it is not possible to know both the position and momentum of a moving particle at the same time.

$$
\Delta \boldsymbol{x} \cdot \boldsymbol{m} \Delta \boldsymbol{u} \geq \frac{\boldsymbol{h}}{4 \pi} \quad \begin{aligned}
& x=\text { position } \\
& u=\text { speed }
\end{aligned}
$$

The more accurately we know the speed, the less accurately we know the position, and vice versa.

## The Quantum-Mechanical Model of the Atom

The matter-wave of the electron occupies the space near the nucleus and is continuously influenced by it.

The Schrödinger wave equation allows us to solve for the energy states associated with a particular atomic orbital.

The square of the wave function gives the probability density, a measure of the probability of finding an electron of a particular energy in a particular region of the atom.


Figure 7.15
Electron probability density in the ground-state H atom.


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## Quantum Numbers and Atomic Orbitals

An atomic orbital is specified by three quantum numbers.
The principal quantum number ( $\boldsymbol{n}$ ) is a positive integer. The value of $n$ indicates the relative size of the orbital and therefore its relative distance from the nucleus.

The angular momentum quantum number ( $l$ ) is an integer from 0 to ( $n-1$ ).
The value of $l$ indicates the shape of the orbital.

The magnetic quantum number $\left(\boldsymbol{m}_{l}\right)$ is an integer with values from $-l$ to $+l$
The value of $m_{l}$ indicates the spatial orientation of the orbital.

## Table 7.2 The Hierarchy of Quantum Numbers for Atomic Orbitals



Sample Problem 7.5 Determining Quantum Numbers for an Energy Level

PROBLEM: What values of the angular momentum $(l)$ and magnetic $\left(m_{l}\right)$ quantum numbers are allowed for a principal quantum number $(n)$ of 3 ? How many orbitals are allowed for $n=3$ ?

PLAN: Values of $l$ are determined from the value for $n$, since $l$ can take values from 0 to ( $n-1$ ). The values of $m_{l}$ then follow from the values of $l$.

SOLUTION: $\quad$ For $n=3$, allowed values of $l$ are $=0,1$, and 2

$$
\begin{array}{|l|}
\hline \text { For } l=0 \quad m_{l}=0 \quad \text { For } l=1 m_{l}=-1,0, \text { or }+1 \\
\hline
\end{array}
$$

For $l=2 m_{l}=-2,-1,0,+1$, or +2
There are $9 m_{l}$ values and therefore 9 orbitals with $\boldsymbol{n}=3$.

Sample Problem 7.6 Determining Sublevel Names and Orbital Quantum Numbers
PROBLEM: Give the name, magnetic quantum numbers, and number of orbitals for each sublevel with the following quantum numbers:
(a) $n=3, l=2$
(b) $n=2, l=0$
(c) $n=5, l=1$
(d) $n=4, l=3$

PLAN: Combine the $n$ value and $l$ designation to name the sublevel. Knowing $l$, we can find $m_{l}$ and the number of orbitals.

## SOLUTION:

|  | $n$ | $l$ | sublevel name | possible $m_{l}$ values \# of orbitals |  |
| :--- | :--- | :--- | :---: | :---: | :---: |
| (a) | 3 | 2 | $3 d$ | $-2,-1,0,1,2$ | 5 |
| (b) | 2 | 0 | $2 s$ | 0 | 1 |
| (c) | 5 | 1 | $5 p$ | $-1,0,1$ | 3 |
| (d) | 4 | 3 | $4 f$ | $-3,-2,-1,0,1,2,3$ | 7 |
| (0 |  |  |  |  |  |

## Sample Problem 7.7 Identifying Incorrect Quantum Numbers

PROBLEM: What is wrong with each of the following quantum numbers designations and/or sublevel names?

|  | $n$ | $l$ | $m_{l}$ | Name |
| :--- | :--- | :--- | :--- | :--- |
| (a) | 1 | 1 | 0 | $1 p$ |
| (b) | 4 | 3 | +1 | $4 d$ |
| (c) | 3 | 1 | -2 | $3 p$ |

## SOLUTION:

(a) A sublevel with $n=1$ can only have $l=0$, not $l=1$. The only possible sublevel name is 1 s .
(b) A sublevel with $l=3$ is an $f$ sublevel, to a $d$ sublevel. The name should be $4 f$.
(c) A sublevel with $l=1$ can only have $m_{l}$ values of $-1,0$, or +1 , not -2 .

Figure 7.16
The $1 s, 2 s$, and $3 s$ orbitals.

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Figure 7.17 The $2 p$ orbitals.


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Figure 7.18 The $3 d$ orbitals.



B Cross section of electron cloud depiction



D Stylized probability contour


E

Figure 7.18

F

G

H

I The five $d$ orbitals

Figure $7.19 \quad$ The $4 f_{\mathrm{xyz}}$ orbital, one of the seven $4 f$ orbitals.


Figure 7.20 Energy levels of the H atom.


